Chapter 5 – Simplifying Formulas and Solving Equations

Look at the geometry formula for Perimeter of a rectangle \( P = L + W + L + W \). Can this formula be written in a simpler way? If it is true, that we can simplify formulas, it can save us a lot of work and make problems easier. How do you simplify a formula?

A famous formula in statistics is the z-score formula \( z = \frac{x - \mu}{\sigma} \). But what if we need to find the \( x \)-value for a z-score of -2.4? Can we back solve the formula and figure out what \( x \) needs to be?

These are questions we will attempt to answer in chapter 5. We will focus on simplifying expressions and solving equations.

Section 5A – Simplifying Formulas and Like Terms

The key to simplifying formulas, is to understand “Terms”. A “term” is a product of numbers and or letters. A term can be a number by itself, a letter by itself, or a product of letters and numbers. Here are some examples of terms:

12b
-11xy
W
-4
\( 7x^2 \)

As you can see, a term has two parts: a numerical coefficient (number part) and most of the time a variable part (letter). Let’s see if we can separate these terms into their numerical coefficients and variable part.

12b : We see that 12 is the numerical coefficient and b is the variable (letter) part
-11xy: We see that -11 is the numerical coefficient and xy is the variable (letter) part
W: This is an interesting term as we don’t see a number part. There is a number part though. Since $W = 1W$, we see that 1 is the numerical coefficient and of course $W$ is the variable (letter) part.

-4: This is also an interesting case as there is no variable part. This is a special term called a constant or constant term. Constants have a number part (-7) but no variable part.

$7x^2$: We see that 7 is the numerical coefficient and $x^2$ is the variable (letter) part.

The degree of a term is the exponent on the variable part. So since $W = W^1$, $W$ is a first degree term. Since $7x^2$ has a square on the variable, this is a second degree term. Notice the number does not influence the degree of a term. A constant like -4 has no variable so it is considered degree zero. Products of letters are tricky. We add the degree of all the letters. So since $3a^2bc = 3a^2b^1c^1$, the degree is $2+1+1 = 4$. It is a 4th degree term.

Try the following examples with your instructor. For each term, identify the numerical coefficient and the variable part (if it has one). Also give the degree of the term.

Example 1: $-8z^3$

Example 2: $r^2$

Example 3: $-15$

One of the key things to know about terms is that we can only add or subtract terms with the same variable part. So we can only add or subtract $x$ with $x$ and $r^2$ with $r^2$ and so on. Terms with the same variable part are called “like terms.” To add or subtract like terms we add or subtract the numerical coefficients and keep the variables (letters) the same.

Look at the example of $5a + 3a$. Are these like terms? They both have the exact same letter part, so they are like terms. Think of it like 5 apples plus 3 apples. We would have 8 apples, not 8 apples squared or 8 double apples. So $5a + 3a = 8a$. We can combine the like terms and keep the letter part the same.
Look at the example of $7a + 2b$. Are these like terms? Since they have different letter parts, they are not like terms. Hence we cannot add them. Think of it like 7 apples plus 2 bananas. That will not equal 9 apple/bananas. It is just 7 apples and 2 bananas. That is a good way of thinking about adding or subtracting terms that don’t have the same letter part. Hence $7a + 2b = 7a + 2b$. They stay separate. In fact, many formulas have two or more terms that cannot be combined. $7a + 2b$ is as simplified as we can make it.

We even have special names for formulas that tell us how many terms it has. A formula with only 1 term is called a “monomial”. A formula with exactly 2 terms is called a “binomial”. A formula with exactly 3 terms is called a “trinomial”.

**Try and simplify the following formulas with your instructor. After it is simplified, count how many terms the simplified form has. Then name the formula as a monomial, binomial or trinomial.**

Example 4: $5w - 8w$

Example 5: $4m + 9$

Example 6: $-3p + -9q + 5p - 4q$

Example 7: $6x^2 + 8x - 14$

**Practice Problems Section 5A**

For each term, identify the numerical coefficient and the variable part (if it has one). Also give the degree of the term.

1. $9L$  
2. $-3r^4$  
3. 18  
4. $y^3$  
5. $-r^2$  
6. $-12p$  
7. $23v^5$  
8. $x^7$  
9. $5^2$
Simplify the following formulas by adding or subtracting the like terms if possible. Count how many terms the simplified form has and then name the formula as a monomial, binomial or trinomial.

25. \(-3a + 11a\)  
26. \(12m + 7m\)  
27. \(-6v - (-14v)\)  
28. \(5x - 14x\)  
29. \(17m + 9m - 8m\)  
30. \(-13p + -9p + 5p\)  
31. \(3y - 8\)  
32. \(6a + 4b + 9b\)  
33. \(-2p + -8p - 3m\)  
34. \(8v - 17v - 12v\)  
35. \(4a + 6b - 8c\)  
36. \(3g - 7h + 10g - 5h\)  
37. \(5w - 8x + 3y\)  
38. \(x^2 + 9x^2 - 7x + 1\)  
39. \(-2w^2 + 4w - 8w\)  
40. \(x^3 + 7x - 9\)  
41. \(4m^3 + 9m^2 - 10m^3 - 7m^2\)  
42. \(5y - 7y^2 + 8y - 3\)

Section 5B – Multiplying Terms with Associative and Distributive Properties

Remember a term is a product of numbers and/or letters. A term can be a letter by itself or a number by itself or a product of numbers and letters. The key thing to remember about terms is that we can only add or subtract “like” terms, that is ones that have the exact same letter part.

Many formulas have terms that cannot be added together. For example the perimeter of a rectangle is \(L + W + L + W\) which when simplified gives \(2L + 2W\). Suppose we want to triple the size of a rectangle. This would triple the perimeter \(3(2L + 2W)\). How would we simplify this formula? This is what we will try to figure out in this section.

Let’s start by thinking about multiplying terms. For example, look at \(4(5y)\). The associative property says that \(a(bc) = (ab)c\). This means that \(4(5y) = (4 \times 5)y = 20y\). Notice we cannot add or subtract unlike terms, but we can always multiply terms since they are a product. The key is just to multiply the number parts together. Look at \((7a)(3b)\). This is the same as \(7 \times 3 \times a \times b\) or \(21ab\). Notice also that when we multiply two terms, our answer is just one term.
Multiply the following terms with your instructor.

Example 1: \(-6(7x)\)

Example 2: \(9a(-4b)\)

Example 3: \((-8m)(-6n)(2p)\)

Let’s look at the example of tripling the perimeter of a rectangle: 3(2L + 2W). The problem again is that 2L and 2W are not like terms so they cannot be added. So how do we multiply by 3? The answer is by using the distributive property. The distributive property is \(a(b+c) = ab+ac\). So to multiply 3(2L + 2W), we multiply 3 times 2L and 3 times 2W. Hence \(3(2L+2W) = 3 \times 2L + 3 \times 2W = 6L + 6W\). So the formula for triple the perimeter is 6L+6W.

Let’s try another example. \(-4(2a-7b)\). It is often helpful to rewrite subtraction as adding a negative. In this example \(-4(2a-7b) = -4(2a + -7b)\). Now we use the distributive property to multiply. \(-4(2a-7b) = -4(2a + -7b) = -4 \times 2a + -4 \times -7b = -8a + +28b\). So the simplified formula is \(-8a + 28b\).
Sometimes we want to subtract a parenthesis. For example, look at \(-3x + 8\). The key is to subtract all the terms inside the parenthesis. This is often called distributing the negative. You can also think of it like multiplying by -1. When we distribute the negative we get the following: 
\[-(3x + 8) = -3x + (-8) = 3x - 8\]. Notice that when we distribute the negative all the signs of the terms inside the parenthesis have changed to the opposite. -3x became +3x and +8 became -8.

Let’s look at a more complicated example of simplifying. \(-6(y - 4) - 5(-2y + 8)\). In problems like this, order of operations comes into play. In order of operations, we simplify parenthesis first. 9y-4 and -2y+8 are both as simplified as possible. They are not like terms so we cannot add or subtract them. Next we do the multiplications. It is very helpful to rewrite subtractions as adding the negative. So \(-6(y - 4) - 5(-2y + 8) = -6(y + -4) + -5(-2y + 8)\). Now we use the distributive property to multiply.
\[-6(y + -4) + -5(-2y + 8) = -6\times y + -6\times -4 + -5\times -2y + -5\times +8 = 54y + 24 + 10y + 40\]

We are not quite finished since now there are some like terms we can add. Remember the y terms with the y terms and the constants with the constants.

\[-54y + +24 + +10y + -40 = -44y + -16\]

So when simplified completely this formula is -44y-16. Remember adding -16 is the same as subtracting 16.

**Simplify the following with your instructor by using the distributive property. Remember to simplify completely.**

**Example 4:** \(7(y + 6)\)

**Example 5:** \(-8(3m - 5n)\)
Example 6: \(-2(x+3w-7y)\)

Example 7: \(5(x+9) - (-8x-11)\)

**Practice Problems Section 5B**

Simplify the following using the distributive property. Be sure to simplify completely.

1. \(5(a+3)\)
2. \(-2(y-9)\)
3. \(4(-3b+4)\)
4. \(-3(-10x-6)\)
5. \(12(3v-8)\)
6. \(-11(3x+5y)\)
7. \(6(2y+1)\)
8. \(-4(2a-7b)\)
9. \(9(2x-4)\)
10. \(-12(-5d-9)\)
11. \(11(4w-12)\)
12. \(-18(3v+5w)\)
13. \(7(a+b+4)\)
14. \(-2c(-3d+13)\)
15. \(7(3x+y-6)\)
16. \(-3p+18\)
17. \(-(-14a+5b-1)\)
18. \(-(8g-6h-4)\)
19. \(6(3y+12)-14y\)
20. \(-2(5a-9b)+9a-4b\)
21. \(7(2x-10)+66\)
22. \(-12(-2d-3)-23d-34\)
23. \(13(3w-11)-(24w+20)\)
24. \(-8(v+3w)-(5v-14)\)
25. \(7(a+b+4)+2ab\)
26. \(-2c(-7d+13)+26c\)
27. \(7(x+y)+2(x-y)\)
28. \(4p-(3p+18)\)
29. \(a+5b-(-14a+5b-1)\)
30. \(-7(-4g+2h+1)-(8g-6h-4)\)
Section 5C – Solving Equations with the Addition Property

Solving equations is a useful tool for determining quantities. In this section we are going to explore the process and properties involved in solving equations.

For example, in business we look at the break-even point. This is the number of items that need to be sold in order for the company’s revenue to equal the cost. This is the number of items that must be sold so that the company is not losing money and is therefore starting to turn a profit. For example a company that makes blue-ray DVD players has costs equal to 40x+12000 where x is the number of blue-ray DVD players made. The equipment needed to make the DVD players was $12000 and it costs about $40 for the company to make 1 DVD player. They sell the DVD players for $70 so their revenue is 70x where x is the number of players sold. The break-even point will be where costs = revenue (40x + 12000 = 70x). How many DVD players do they need to sell to break even?

To solve problems like this we need to learn how to solve equations like 40x + 12000 = 70x. To solve an equation, we are looking for the number or numbers we can plug in for the variable that will make the equation true. For example, try plugging in some numbers for x in the break-even equation and see if it is true. If we plug in 100, we get the following: 40(100) + 12000 = 70(100). But that is not true! 4000 + 12000 ≠ 7000 so 100 is not the solution. If we plug in 400 we get the following: 40(400) + 12000 = 70(400). This is true since 16000 + 12000 = 28000. The two sides are equal! So the company needs to make and sell 400 blue-ray DVD players to break even. After 400, they will start to turn a profit.

As you can see sometimes we can guess the answer to an equation. If you cannot guess the answer, then we need to have ways of figuring out the answer.

3 Types of Equations

There are 3 types of equations: conditional, contradiction and identity. The break-even point equation has only 1 solution. This type of equation is conditional because it is only true if x = 400 and is not true if x is any other number. When an equation has a finite number of solutions, it is conditional. Some equations like w+3 = w+5 are never true. No matter what number we replace w with the equation is never true. When an equation has no solution it is called a contradiction equation. The third type of equation is an identity equation. This is one that is always true. Look at y+4 = y+4. We can plug in any number we want for y and it will be true. For example we could plug in 70 and see that 70+4=70+4 (true). We could plug in -549 and see that -549+4 = -549+4. Every time we plug in any number we get a true statement. The solution to an identity equation is “all real numbers”, since it has infinitely many solutions.
Addition Property of Equality

How do we find the answer to an equation when we cannot guess the answer? One property that is very helpful is the addition property. The addition property says that we can add or subtract the same number or term to both sides of an equation and the equation will remain true.

For example look at the equation \( w + 19 = -4 \). You may or may not be able to guess the number we can plug in for \( w \) that makes the equation true. The key is to add or subtract something from both sides so that we isolate the variable \( w \). In equation solving, it is all about opposites. Do the opposite of what is being done to your letter. Since we are adding 19 to our variable \( w \), let’s subtract 19. Look what happens if we subtract 19 from both sides and simplify.

\[
w + 19 - 19 = -4 - 19
\]
\[
w + 0 = -4 + -19
\]
\[
w = -23
\]

First notice, that we had to subtract the same number from both sides. If you only subtracted 19 from the left side of the equation, the equation would no longer be true. Also subtracting 19 is the same as adding -19 from both sides. This helps when dealing with negative numbers. This shows us that the number we can plug in for \( w \) that makes the equation true is -23. How can we check if that is the correct answer? We plug in -23 for \( w \) in the original equation and see if the two sides are equal. \(-23 + 19 = -4\) is true so -23 is the correct answer! Notice also that this is a conditional equation and is only true when \( w = -23 \) and false for any other number.

Let’s look at another example equation \( 5x + 7 = 4x - 3 \). When dealing with an equation like this, our goal is to bring letters to one side and the constant numbers to the other side. If we want to eliminate the 4x on the right side, we can subtract 4x from both sides.

\[
5x + 7 = 4x - 3
\]
\[
-4x
\]
\[
x + 7 = 0 - 3
\]
\[
x + 7 = -3
\]

Notice that there are only \( x \) variables on the right side. Can you guess the answer now? If not we can get rid of the 7 by subtracting 7 (adding -7) to both sides.

\[
x + 7 = -3
\]
\[
-7
\]
\[
x + 0 = -10
\]
\[
x = -10
\]
Notice the number we can plug in for x that makes the equation true is -10. Check if that is the correct answer. Plugging in -10 into the original equation we get the following:

\[5(-10) + 7 = 4(-10) - 3\]
\[-50 + 7 = -40 - 3\]
\[-43 = -43\]

So when we plug in -10, we do get a true statement. Hence -10 is the solution. Note that the two sides were equal and both equal to -43, but -43 is not the solution. The solution is the number we replaced the letter with that made the two sides equal. Also notice this was a conditional equation. It was only true when x = -10 and false otherwise.

What about contradiction and identity equations. Remember a contradiction is never true and an identity is always true.

Look at \[3b + 7 = 3b - 2\]. A technique in solving equations is to bring the variable terms to one side. But if we subtract 3b from both sides we get +7 = -2. That is never true!! This tells us that the equation is never true no matter what. Hence this is a contradiction equation and the answer is “No Solution”.

Look at \[5a + 1 = 5a + 1\]. Did you notice the two sides are exactly the same? If not, we can try to bring the variable terms to one side. But if we subtract 5a from both sides we get +1 = +1. That is always true!! This tells us that the equation is true no matter what we plug in for a. Hence this is an identity equation and the answer is “All Real Numbers”.

Try to solve the following equations with your instructor using the addition property. Be sure to check your answers.

Example 1: \[w - 8 = -15\]

Example 2: \[6d + 8 = 5d - 3\]
Example 3: \( 3y + 4 = 3y \)

Example 4: \( 2d + 7 = 5d - 3d + 7 \)

Example 5: \( w - \frac{1}{6} = \frac{2}{3} \)

Example 6: \( 5.7p + 0.35 = 4.7p - 0.56 \)
Practice Problems Section 5C

Solve the following equations with the addition property.

1. \( x + 9 = 23 \)
2. \( m - 12 = -5 \)
3. \( y - 6 = -17 \)
4. \( 13 = v + 7 \)
5. \( -18 = w - 15 \)
6. \( h - 13 = -24 \)
7. \( m + 0.13 = 0.58 \)
8. \( n - \frac{1}{2} = \frac{3}{8} \)
9. \( -1.19 = -2.41 + T \)
10. \( \frac{4}{5} = \frac{2}{7} + c \)
11. \( x - 74 = -135 \)
12. \( y + -53 = -74 \)
13. \( c - 8.14 = 6.135 \)
14. \( d + \frac{1}{5} = \frac{3}{4} \)
15. \( -630 = -440 + p \)
16. \( 39 = y + 86 \)
17. \( n + 0.0351 = -0.0427 \)
18. \( -\frac{1}{10} = \frac{3}{4} + L \)
19. \( 6a = 5a - 7 \)
20. \( 8b - 13 = 8b + 2 \)
21. \( 17v = 16a - 9 \)
22. \( -8b = -9b + 4 \)
23. \( -8x + 8x = 13 - 13 \)
24. \( -5w = -6w + 11 \)
25. \( -9a - 13 = -10a + 4 \)
26. \( 1.3x - 2.7 = 0.3x - 3.4 \)
27. \( 16w + 9 - 15w = w + 9 \)
28. \( -5.2x + 7.3 = -6.2x - 3.6 \)
29. \( 6a - \frac{1}{4} = 5a + \frac{3}{4} \)
30. \( 0.8b - 2.57 = -0.2b + 2.9 \)
31. \( a - 0.004 = -0.5a + 0.053 + 0.5a \)
32. \( \frac{8}{5}x - \frac{1}{3} = \frac{3}{5}x + \frac{1}{3} \)
33. \( 26x + 19 - 21x = 5x - 17 \)
34. \( -5.2d + 7.3 = -6.2d - 3.6 \)
Section 5D – Solving Equations with the Multiplication Property of Equality

Look at the equation $4c = 17$. You probably cannot guess what number we can plug in for the variable that will make the equation true. Also subtracting 4 will not help. If we subtract 4 we will get $4c – 4 = 17 - 4$. That equation is more complicated, not less. This equation requires the multiplication property in order to solve it.

The multiplication property of equality says that we can multiply or divide both sides of the equation by any non-zero number. Remember, the key to equation solving is doing the opposite of what is being done to your variable. In $4c = 17$, the variable is being multiplied by 4, so we should do the opposite. The opposite of multiplying by 4 is dividing by 4. Dividing both sides by 4 gives us the following.

\[
\frac{4c}{4} = \frac{17}{4}
\]

4 divided by 4 is 1 and 17 divided by 4 is 4.25 so we get the following:

\[
4c = 17
\]

\[
\frac{4c}{4} = \frac{17}{4}
\]

\[
c = 4.25
\]

Hence the answer is 4.25. Again we can check it by plugging in 4.25 into the original equation and seeing if it is equal. $4(4.25) = 17$

Let’s look at another example. \( \frac{y}{7} = 2.5 \)

Since we are dividing our variable by 7, we should multiply both sides by 7 in order to isolate the variable.

\[
7\left(\frac{y}{7}\right) = 7(2.5)
\]

\[
\frac{y}{7} \cdot 7 = 7(2.5)
\]

\[
y = 17.5
\]

Since 7/7 = 1 and 7x2.5 = 17.5 and 1y = y, we are left with $y = 17.5$

Let’s look at a third example. \( \frac{-2}{3}w = \frac{7}{8} \)
The variable is being multiplied by \(-\frac{2}{3}\) so we need to divide both sides by \(-\frac{2}{3}\). If you remember from fractions, dividing by a fraction is the same as multiplying by the reciprocal. So dividing by \(-\frac{2}{3}\) is the same as multiplying by \(-\frac{3}{2}\). So we are going to multiply both sides by \(-\frac{3}{2}\).

\[
\frac{-3}{2} \left( -\frac{2}{3} w \right) = \frac{-3}{2} \left( \frac{7}{8} \right)
\]

Notice the reciprocals multiply to positive 1. So we are left with an answer of \(-\frac{21}{16}\) or \(-\frac{5}{16}\).

\[
\frac{-3}{2} \left( -\frac{2}{3} w \right) = \frac{-3}{2} \left( \frac{7}{8} \right)
\]

\[
\frac{6}{6} w = -\frac{21}{16}
\]

\[
w = -\frac{21}{16}
\]

\[
w = -1 \frac{5}{16}
\]

Solve the following equations with your instructor by using the multiplication property.

Example 1: \(-8b = 168\)

Example 2: \(\frac{x}{17} = -3\)
Example 3: \(-\frac{3}{5}w = \frac{1}{4}\)

**Practice Problems Section 5D**

Solve the following equations. Simplify all fractions completely.

1. \(2x = 22\)
2. \(28 = -4y\)
3. \(12m = -72\)
4. \(-8 = 40n\)
5. \(-9w = -144\)
6. \(-65 = -5y\)
7. \(6m = 90\)
8. \(-98 = -7a\)
9. \(14v = -70\)
10. \(13 = -39n\)
11. \(-120d = -20\)
12. \(180 = -15x\)
13. \(34 = -2h\)
14. \(12g = 240\)
15. \(-51 = -3L\)
16. \(14f = -70\)
17. \(5 = 20x\)
18. \(36c = -4\)
19. \(\frac{2}{7}y = \frac{3}{5}\)
20. \(\frac{4}{5} = -\frac{12}{25}u\)
21. \(-\frac{1}{9}u = -\frac{5}{18}\)
22. \(\frac{4}{9} = -7y\)
23. \(3b = -\frac{1}{5}\)
24. \(-\frac{4}{11} = \frac{16}{33}L\)
25. \(-\frac{3}{8}T = \frac{9}{8}\)
26. \(-\frac{3}{14} = -\frac{3}{7}h\)
27. \(\frac{6}{13}p = -\frac{14}{13}\)
28. \(0.4m = 5.2\)
29. \(0.3 = -0.24a\)
30. \(1.2w = -0.144\)
31. \(1.8 = -0.09p\)
32. \(-0.1d = 0.037\)
33. \(-0.47 = -2.35x\)
34. \(6.156 = -1.8n\)
35. \(0.004c = -0.2\)
36. \(0.2312 = 6.8b\)
37. \(0.035g = -0.056\)
38. \(-12 = 0.05f\)
39. \(-0.33m = -8.58\)
Section 5E – Steps to Solving General Linear Equations

A linear equation is an equation where the variable is to the first power. If a variable has an exponent of 2 (square) or 3 (cube) or higher, then it will require more advanced methods to solve the problem. In this chapter we are focusing on solving linear equations.

We have seen that we can solve equations by guessing the answer. If we cannot guess we can use the multiplication and addition properties to help us figure out the answer. Let’s now look at some more complicated equations and the steps to solving them.

Steps to Solving a Linear Equation (It is Vital to Memorize These!!)

1. Eliminate parenthesis by using the distributive property.
2. Eliminate fractions by multiplying both sides of the equation by the LCD.
3. Eliminate decimals by multiplying both sides of the equation by a power of 10 (10, 100, 1000...)
4. Use the addition property to eliminate variable terms so that there are only variables on one side of the equation.
5. Use the addition property to eliminate constants so that there are only constants on one side of the equation. The constants should be on the opposite side of the variables.
6. Use the multiplication property to multiply or divide both sides of the equation in order to isolate the variable by creating a coefficient of 1 for the variable.
7. Check your answer by plugging it into the original equation and see if the two sides are equal.

Note: After each step, always add or subtract like terms that lie on the same side of the equation.

Note: Remember that an equation can have “no solution” or “all real numbers” as a solution in the cases of contradiction and identity equations.
Let’s look at an example \(-6(2w-8)+4w=3w-(7w+9)\)

Step 1: Our first step is to distribute and eliminate parenthesis, so we will multiply the -6 times both the 2w and the -8. We will also distribute the negative to the 7w and the 9 and eliminate that parenthesis as well. We should only distribute to the terms in the parenthesis. For example we do not distribute the -6 to the 4w since the 4w is not in the parenthesis.

\[-6(2w-8)+4w=3w-(7w+9)\]
\[-12w+48+4w=3w-7w-9\]

Always look to simplify after each step. For example in this problem the -12w and 4w are like terms on the same side. Also the 3w and -7w are also like terms on the same side. If you struggle with negatives, you can convert the -7w to adding the opposite. Be careful. **Do not add or subtract terms on opposite sides of the equation.**

\[-6(2w-8)+4w=3w-(7w+9)\]
\[-12w+48+4w=3w-7w-9\]
\[-8w+48 = -4w - 9\]

Step 2 and 3: There are no fractions or decimals so we proceed directly to step 4.

Step 4: We need to bring the variables to one side. You can bring variables to either side, but many students like to bring the variables to the left side only. So we will need to eliminate the -4w on the right side. Hence we will add the opposite +4w to both sides.

\[-8w+48 = -4w-9\]
\[+4w + 4w\]
\[-4w+48 = 0-9\]
\[-4w+48 = -9\]

Step 5: We need to bring the constants to the opposite side as the variables. So we need to get rid of the +48. Hence we will subtract 48 (add -48) to both sides. We are then left with

\[-4w = -57\]
\[-4w+48 = -9\]
\[-48 - 48\]
\[-4w+0 = -57\]
\[-4w = -57\]
Step 6: We now need to get the $w$ by itself. Since the $w$ is being multiplied by $-4$, we will divide both sides by $-4$ to get our answer of $57/4$. Notice the answer can be written three ways and all are equally correct \[
\left( \frac{57}{4} or 14.25 \right)
\]

\[-4w = -57
\]
\[
\frac{1}{4}w = \frac{-57}{-4}
\]
\[
w = 14.25
\]

Step 7: Let’s check our answer by plugging into the original equation. Notice that all of the $w$’s have to be replaced with $14.25$ and don’t forget to use the order of operations when simplifying each side. As you can see, checking your answer can be just as much work as solving the equation.

\[-6(2w-8)+4w = 3w-(7w+9)
\]
\[-6(2\times14.25-8)+4\times14.25 = 3\times14.25-(7\times14.25+9)
\]
\[-6(28.5-8)+4\times14.25 = 3\times14.25-(99.75+9)
\]
\[-6(20.5)+4\times14.25 = 3\times14.25-(108.75)
\]
\[-123+57 = 42.75-(108.75)
\]
\[-66 = -66
\]

Let’s try another example. Look at $\frac{1}{3}c - \frac{3}{5} = \frac{1}{2}c + 4$

Step 1: There are no parenthesis so we proceed to step 2.

Step 2: To eliminate fractions we find the LCD. Since the denominators are 3, 5 and 2 the LCD is 30. Hence we will multiply everything on both sides by 30. This will eliminate the fractions. Remember all the terms must be multiplied by 30. When multiplying a whole number (30) by fractions it is good to write the whole number as a fraction (30/1).

\[\frac{30}{1} \left( \frac{1}{3}c - \frac{3}{5} \right) = \frac{30}{1} \left( \frac{1}{2}c + 4 \right)
\]
\[\frac{30}{1} \times \frac{1}{3}c - \frac{30}{1} \times \frac{3}{5} = \frac{30}{1} \times \frac{1}{2}c + \frac{30}{1} \times 4
\]
\[\frac{30}{1}c - \frac{90}{5} = \frac{30}{2}c + \frac{120}{1}
\]
\[
10c - 18 = 15c + 120
\]
Notice we are now left with an equation without fractions.

Step 3: There are no decimals, so we proceed to step 4.

Step 4: We bring all the variables to the left side by subtracting 15c (adding -15c) to both sides.

\[
10c - 18 = 15c + 120 \\
-15c \quad -15c \\
-5c - 18 = 0 + 120 \\
-5c - 18 = 120
\]

Step 5: We bring all the constants to the opposite side. We can eliminate the -18 by adding +18 to both sides.

\[
-5c - 18 = 120 \\
+18 \quad +18 \\
-5c + 0 = 138 \\
-5c = 138
\]

Step 6: Isolate the variable. Since we are multiplying the w by -5, we divide both sides by -5.

\[
-5c = 138 \\
\frac{1}{-5} c = \frac{138}{-5} \\
1c = -\frac{138}{5} \\
c = -27 \frac{3}{5}
\]

Step 7: Check your answer. By plugging -138/5 into the original equation, the two sides are equal.

Let’s try a third example: \(0.24(3m-1) = 0.82m + 0.24 - 0.1m\)

Step 1: We will distribute the 0.24 to the 3m and -1 to eliminate the parenthesis. We will make sure to combine any like terms that are on the same side. Notice 0.82m and -0.1m are like terms on the same side, so we can add them.

\[
0.24(3m-1) = 0.82m - 0.24 - 0.1m \\
0.24 \times 3m + 0.24 \times -1 = 0.82m - 0.24 - 0.1m \\
0.72m - 0.24 = 0.72m - 0.24
\]
Step 2: There are no fractions, so we proceed to step 3.

Step 3: Since the most decimal places to the right is two (hundredths place), we will multiply everything on both sides by 100. If one of the decimals had ended in the thousandths place, we would multiply by 1000 and so on. Notice all the decimals are gone.

\[
0.72m - 0.24 = 0.72m - 0.24 \\
100(0.72m - 0.24) = 100(0.72m - 0.24) \\
100 \times 0.72m + 100 \times -0.24 = 100 \times 0.72m + 100 \times -0.24 \\
72m - 24 = 72m - 24
\]

Step 4: Bring the variables to one side by subtracting 72m from both sides. Notice all variables cancel and we are left with -24 = -24 which is a true statement.

\[
72m - 24 = 72m - 24 \\
-72m \quad -72m \\
0 - 24 \quad 0 - 24 \\
-24 = -24
\]

Since we are left with a true statement and all the variables are gone, we need go no further. This is an always true equation. So the answer is “All Real Numbers”.

**Try to solve the following equations with your instructor.**

Example 1: \(4(3d + 7) = 19(d - 2) - 5d + 2\)
Example 2: \[
\frac{1}{4}L - \frac{1}{2} = \frac{1}{3}L + \frac{5}{6}
\]

Example 3: \[0.25(y + 0.4) + 0.2 = 0.15y - 0.5\] (Remember to eliminate parenthesis before eliminating decimals)

---

**Practice Problems Section 5E**

Solve the following equations. If your answer is a fraction, be sure to simplify it completely. For #19 and #20, remember to eliminate parenthesis before eliminating fractions or decimals.

1. \[7y - 15 = 20\]
2. \[-18 = 4p + 2\]
3. \[12 = -1a - 17\]
4. \[9 + -7n = 8\]
5. \[-12x + 7 = -9x - 11\]
6. \[5y - 14 = -3y + 18\]
7. \[-1a + 27 = -6a - 13\]
8. \[-2b + 13 = 9b + 7\]
9. \[6 - 13y = -5 + 11y\]
10. \[-23 + 5m = -19 - 18m\]
11. \[13 - y = -1y + 17 - 4\]
12. \[14g - 3 = -3g + 8\]
13. \[-31m + 17 + 28m = -3m - 10 + 27\]
14. \[25 + 5n - 24 = -3n + 18 + 8n\]
15. \[-2(4x + 3) = -6x\]
16. \[7(y - 3) + 2y = 6\]
17. \(-4(5a-1)+18a = -2(-3a+2)\) 
18. \(11b-(7b+9) = 4(b-3)\) 
19. \(\frac{1}{2}m+1=-\frac{1}{3}m-2\) 
20. \(\frac{2}{3}n-\frac{1}{5} = \frac{2}{5}\) 
21. \(\frac{3}{4}c+2=1c-\frac{1}{2}\) 
22. \(\frac{1}{6}d-\frac{3}{2} = \frac{1}{2}d + \frac{5}{6}\) 
23. \(\frac{1}{5}(2p+1) = \frac{1}{2}(\frac{1}{3}p-1)\) 
24. \(\frac{2}{3}(w+6) = \frac{1}{4}(w-8)\) 
25. \(0.5x-0.3 = 0.8x+0.9\) 
26. \(0.09p-0.04 = 0.09p+0.06\) 
27. \(0.23x-0.4 = 0.38x+0.2\) 
28. \(0.007y-0.03 = 0.009y+0.06\) 
29. \(0.2m+0.41 = 0.18m+0.67\) 
30. \(0.6n-2 = 1y+0.3\) 
31. \(-0.4(2a+1) = -0.9a+0.5+0.1a-0.9\)

**Section 5F – Solving Proportions**

A special type of equation that has many applications are proportions. A “proportion” is when two fractions are equal to each other. For example \(\frac{3}{5} = \frac{6}{10}\). Notice the two fractions are equal to each other. One way to check if two fractions are equal is by looking at the cross products. Notice the numerator of one fraction (3) times the denominator of the other fraction (10) is equal to the other cross product (6x5). Notice both cross products are equal to 30. This is an easy way to check if two fractions are equal.

In general if \(\frac{c}{d} = \frac{e}{f}\) then \(c \times f = e \times d\).

Try the following examples with your instructor.

Example 1: Are these two fractions equal? \(\frac{3}{7} = \frac{9}{22}\)
Example 2: Are these two fractions equal? \( \frac{8}{18} = \frac{12}{27} \)

Often we need to solve a proportion though. Setting the cross products equal is an easy way to simplify a proportion equation and make it much easier to solve.

Let’s look at an example. Solve the following proportion: \( \frac{15}{16} = \frac{h}{8} \)

First we notice that this is two fractions set equal, so it is a proportion. If it does not have this form, you need to use the methods discussed in section 5D. Since this is a proportion we can set the cross products equal to each other.

\[
\frac{15}{16} = \frac{h}{8} \\
15 \times 16 = 15 \times 8 \\
h \times 16 = 15 \times 8 \\
\]

Simplifying gives us \(16h = 120\). Now we solve by isolating the variable (divide both sides by 16).

\[
16h = 120 \\
\frac{16h}{16} = \frac{120}{16} \\
1h = 7.5 \\
h = 7.5 \\
\]

At the beginning of this chapter, we talked about the famous z-score formula in statistics. \( z = \frac{x - \mu}{\sigma} \). Can we find the x-value (pounds) for a z-score of -2.4? If \( \mu = 9.5 \) and \( \sigma = 1.5 \) can we back solve the formula and figure out what x needs to be?

Plugging in the correct values into the formula gives us the following:
\[ z = \frac{x - \mu}{\sigma} \]
\[ -2.4 = \frac{x - 9.5}{1.5} \]

Now if we were to write -2.4 as -2.4/1 we would have a proportion! We could then cross multiply and solve. Let’s try it.

\[ z = \frac{x - \mu}{\sigma} \]
\[ -2.4 = \frac{x - 9.5}{1.5} \]
\[ -2.4 = \frac{x - 9.5}{1.5} \]
\[ -2.4 = \frac{x - 9.5}{1.5} \]

Setting the cross products equal and solving gives us the x value.

\[ \frac{-2.4}{1} = \frac{x - 9.5}{1.5} \]
\[ 1(x - 9.5) = -2.4(1.5) \]
\[ x - 9.5 = -3.6 \]
\[ +9.5 \quad +9.5 \]
\[ x + 0 = 5.9 \]
\[ x = 5.9 \]

So the x value for a z-score of -2.4 is 5.9 pounds.

Try the following proportion problems with your instructor:

Example 1: \[ \frac{3.5}{b} = \frac{2}{5} \]
Example 2: \[ \frac{y+6}{7} = \frac{y-4}{3} \]

Practice Problems Section 5F

Solve the following proportion problems. Simplify all fraction answers completely.

1. \[ \frac{1}{x} = \frac{3}{5} \]
2. \[ \frac{w}{-4} = \frac{1}{6} \]
3. \[ \frac{-3}{7} = \frac{y}{4} \]
4. \[ \frac{8}{5} = \frac{3}{2d} \]
5. \[ \frac{7.5}{x} = \frac{2.5}{9} \]
6. \[ \frac{3w}{7} = \frac{2}{3} \]
7. \[ \frac{-4}{13} = \frac{-1}{m} \]
8. \[ \frac{-11}{x} = \frac{22}{5} \]
9. \[ \frac{8}{2.1} = \frac{w}{6.3} \]
10. \[ \frac{1}{5w} = \frac{-8}{15} \]
11. \[ \frac{0.25}{4} = \frac{0.15}{L} \]
12. \[ \frac{-18}{25} = \frac{9}{-5u} \]
13. \[ \frac{0.4}{1.2} = \frac{-0.8d}{2.4} \]
14. \[ \frac{-50}{3h} = \frac{20}{9} \]
15. \[ \frac{3}{2.5} = \frac{1.8f}{7.5} \]
16. \[ \frac{2p-1}{5} = \frac{3p}{10} \]
17. \[ \frac{7}{x+4} = \frac{2}{x+9} \]
18. \[ \frac{3w}{7} = \frac{2w+5}{4} \]
19. \[ \frac{-5}{m-1} = \frac{-2}{m+3} \]
20. \[ \frac{x+4}{6} = \frac{2x}{11} \]
21. \[ \frac{w-7}{1.5} = \frac{w}{1.5} \]
22. \[ \frac{6}{x+5} = \frac{4}{x-7} \]
23. \[ \frac{w-14}{9} = \frac{3w}{17} \]
24. \[ \frac{3}{1.5+k} = \frac{7}{4.5} \]
Chapter 5 Review

In chapter 5, we looked at simplifying algebraic expressions by adding and subtracting like terms and by using the associative and distributive properties. Remember like terms have the exact same variable part. We cannot add $7f$ to $5w$. They are not like. If the terms are like, then we can add or subtract the numerical coefficients and keep the variable the same. ($8p - 3p = 5p$) We can always multiply terms by simply multiplying the numerical coefficients and putting the variables together ($7a \times 8b = 56ab$). When we want to multiply a number or term times a sum or difference, we need to use the distributive property like $3(4v + 5w) = 12v + 15w$.

We also looked at solving equations. Remember the solution to an equation is the number or numbers that make the equation true. For example the solution to $3n + 1 = 13$ is $n = 4$ because when we plug in 4 for $n$ we get $3(4)+1 = 12+1 = 13$ (a true statement). Some equations have no solution and some equations have a solution of All Real Numbers.

The steps to solving equations are critical to remember. Here are the steps again in order. Remember that after each step, always add or subtract like terms that lie on the same side of the equation.

**Steps to Solving a Linear Equation**

1. Eliminate parenthesis by using the distributive property.

2. Eliminate fractions by multiplying both sides of the equation by the LCD.

3. Eliminate decimals by multiplying both sides of the equation by a power of 10 (10, 100, 1000...)

4. Use the addition property to eliminate variable terms so that there are only variables on one side of the equation.

5. Use the addition property to eliminate constants so that there are only constants on one side of the equation. The constants should be on the opposite side of the variables.

6. Use the multiplication property to multiply or divide both sides of the equation in order to isolate the variable by creating a coefficient of 1 for the variable.

7. Check your answer by plugging it into the original equation and see if the two sides are equal.

Finally, we looked at two equal fractions called a proportion. We say that we can solve a proportion by setting the cross products equal to each other and solving.
Chapter 5 Review Problems

Simplify the following algebraic expressions. Tell how many terms the answer has and label it as monomial (1 term), binomial (2 terms), trinomial (3 terms) or multinomial (4 or more terms).

1. \(-3c + 7c - 8c\)  
2. \(6a - 9b + 11a - 3b\)  
3. \(7(5cd)\)  
4. \(\frac{1}{2}(6w)\)  
5. \(-7(4x - 9)\)  
6. \(3a(b + 4)\)  
7. \(-5(2g + 7) - 3g + 19\)  
8. \(3h + 19 - (-2h + 7)\)  
9. \(-2(4q + 7) - (3q - 8)\)

Solve the following equations. Simplify fraction answers completely.

10. \(3x + 6 = 12\)  
11. \(9 - 4y = 7\)  
12. \(-3z + 6 = -4z - 8\)  
13. \(-7c + 3 + 5c = 2 + 3c + 8\)  
14. \(7(9a - 2) = 63a + 8\)  
15. \(-6(d - 3) = d - 7d + 18\)  
16. \(-\frac{1}{3}w + 1 = \frac{1}{2}w + \frac{1}{2}\)  
17. \(\frac{1}{5}y - \frac{2}{3} = \frac{1}{3}y + \frac{1}{5}\)  
18. \(-\frac{3}{4}p + 2 = -\frac{1}{2}p + \frac{5}{4}\)  
19. \(-\frac{3}{5}v - \frac{1}{4} = \frac{3}{5}v - \frac{3}{4}\)  
20. \(0.45x - 0.9 = 0.35x + 0.4\)  
21. \(0.08y + 0.012 = -1.92y - 0.034\)  
22. \(0.05a + 1.9 = 0.03x - 0.4\)  
23. \(1.5b + 3 = -2.5b - 7\)  
24. \(0.04(p + 3) = 0.15p + 0.12 - 0.11p\)  
25. \(\frac{2}{3}(2x + 1) = \frac{1}{6}x\)

Solve the following proportions. Simplify fraction answers completely.

26. \(\frac{-4}{w} = \frac{3}{8}\)  
28. \(\frac{3}{F + 4} = \frac{5}{F - 1}\)  
29. \(\frac{6}{-7} = \frac{g + 4}{2}\)  
30. \(\frac{-2w - 1}{5} = \frac{w + 1}{4}\)